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THE EFFECT OF LONGITUDINAL SLOPE ON SOLIDS TRANSPORT AND FRICTION IN PARTICLE-LADEN FLOW ABOVE STATIONARY DEPOSIT IN PIPE

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The effect is studied of the longitudinal slope of the top of a stationary bed on bed transport and friction in flow carrying solid particles above the bed deposit in a pressurized pipe. The analysis is based on results of our experiments carried out in a 100 mm pipe loop with an inclinable invert U-tube section. In the literature, various versions of formulae can be found for bed load transport. However, all of them are calibrated for very mild longitudinal slopes as they expect an application to conditions in open channels where the slope is small and its variation marginal. In our analysis, we employ our versions of the Meyer-Peter and Müller (MPM) transport formula and of the bed friction formula for the intense transport condition. The aim is to use our experimental results for inclined flows including those at very steep inclinations to examine the implementation of the effect of the longitudinal slope in the predictive formulae. Applications of the presented research results include sediment transport and morphology of mountain streams as well as hydraulic transport of solids in inclined pipes. A layered model for settling slurry flows in inclined pipes uses the transport- and friction formulae to predict the energy head loss and internal structure of the slurry flow.

KEY WORDS: sheet flow, bed load, pipe experiment

NOTATION

A	cross-sectional area of pipe (m^2)
B_s	constant in friction law for hydraulically-rough boundary
c	local volumetric concentration
C_b, C_a	mean spatial volumetric concentration in the bed, in area above bed respectively
C_{vd}	mean delivered volumetric concentration
C_{vi}	mean spatial volumetric concentration in entire pipe cross section
d_{50}	mass-median diameter of particle (m)
D	inner diameter of pipe (m)
g	gravitational acceleration (m/s^2)
i	hydraulic gradient
k_s	equivalent roughness of the top of the bed (m)
L	length of pipe (m)
O	perimeter (m)

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p	pressure (Pa)
q_s	volumetric transport rate for unit width (m²/s)
Q	volumetric flow rate (m³/s)
Re_p	particle Reynolds number
R_h	hydraulic radius (m)
S	relative density
V	mean velocity of flow (m/s)
w_t	terminal settling velocity of solid particle (m/s)
W_{s*}	grain parameter
y	vertical position above bottom of pipe (m)
α	empirical coefficient in solids transport formula
β	empirical coefficient in solids transport formula
θ	Shields parameter (dimensionless shear stress) for bed
κ	Karman constant
λ	Darcy-Weisbach friction coefficient
ν_f	kinematic viscosity of fluid (m²/s)
ρ	density of fluid (kg/m³)
τ	shear stress at flow boundary (Pa)
Φ	Einstein transport parameter
ω	angle of pipe inclination = angle of longitudinal slope of top of bed

Subscripts:

a, b	area above bed, bed respectively
f, m, s	fluid, mixture, solids
fric, man	frictional, manometric
w	wall
ω	inclined

1. FLOW OVER STATIONARY DEPOSIT

Intense transport of solids associated with high bed shear at the plane top of an erodible stationary bed is typical for flows exhibiting steep slopes of energy grade line. Stratified slurry flows in pressurized pipelines are typical examples of such flows.

1.1 PREVIOUS WORK

Pressurized flows above the stationary bed in horizontal pipes were studied in the past (e.g. Sumer et al., 1996; Pugh and Wilson, 1999; Matoušek, 2009; Matoušek and Krupička, 2014; Matoušek et al., 2014). The studies showed that transport of solids (bed load) and bed friction are interrelated in flows at high bed shear. In order to analyse the mutual relation, it is essential to have information on the distribution of concentration (and if possible also velocity) in mixture flow. Such information is scarce.

Pugh and Wilson (1999) used their information on measured distributions of concentration and velocity for an evaluation of the thickness of the transport layer above the deposit and the equivalent roughness of the top of the deposit and did not proceed to formulate a model for a prediction of the flow structure.

Matoušek (2011) proposed a predictive model for flow above the plane bed at high bed shear in a pipe. It predicts the slope of the energy grade line, i, and the thickness of the deposit, y_b, for flow carrying certain solids at certain V_m and C_{vd} in the pipe of D. The model employs suitable formulae for solids transport above deposit (Matoušek, 2009) and friction at the top of the deposit (Matoušek and Krupička, 2014).

Recently, the formulae were applied in a modified form in a layered model for settling slurry flows in an inclined pipe (Matoušek et al., 2018). Usually, the model is employed to predict inclined stratified slurry flows with sliding beds and it was validated for such flows with experimental data from two laboratory loops (Institute of Hydrodynamics in Prague and Delft University of Technology).

In this paper, we present and discuss the results of an experiment with the inclined flow with a stationary bed. The results are suitable for an evaluation of the inclination-related modifications of the formulae for bed transport and friction.

1.2 PRESSURIZED FLOW IN PIPE

Contrary to standard open channel flow, pipe flow above a mobile bed is significantly affected by the 'side-wall effect', i.e. by the presence of the pipe wall. The cross sectional area of the flow above the bed must be split into two parts, one associated with the top of the bed and the other with the pipe wall (Figure 1). The boundary between the two sub-areas crosses the positions of the maximum local velocities in the flow cross sectional area.

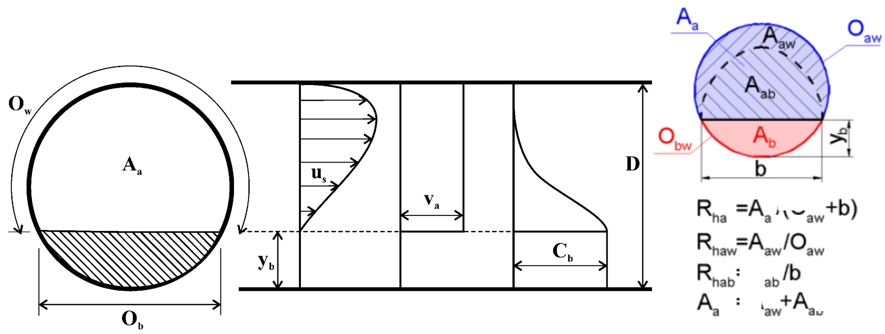

Figure 1. Schematic geometry of pipe cross section and schematic distribution of velocity and concentration in flow above stationary bed (Matoušek 2011)

1.3 BED FORMULAE IN LAYERED MODEL FOR INCLINED FLOWS

The formulae are modified for conditions in inclined flow simply by introducing the gravitational acceleration component normal to the surface of the inclined bed.

The discharge of grains above the bed is determined using a transport formula of Meyer-Peter and Müller (MPM) type. Our version of the MPM-formula has been obtained by integration of the product of local velocities and concentrations of solids over the discharge area of a shear layer (Matoušek, 2011) and it has been calibrated by a large number of experimental data for horizontal flows (Matoušek and Krupička, 2014),

$$\Phi = \left(5.22 + \frac{39}{\mathrm{Re}_p^{0.62}}\right) \cdot \theta^{\left(1.2+\frac{2.6}{\mathrm{Re}_p^{0.39}}\right)} \tag{1}$$

where Einstein transport parameter $\Phi = q_s / \sqrt{(S_s - S_f) \cdot g \cdot \cos\omega \cdot d_{50}^3}$, the sediment volumetric discharge per unit width of bed $q_s = C_a \cdot A_a \cdot V_a / O_b$, the bed Shields parameter $\theta = \tau_b / \left[(\rho_s - \rho_f) \cdot g \cdot \cos\omega \cdot d_{50}\right]$, and the particle Reynolds number $\mathrm{Re}_p = w_t \cdot \cos\omega \cdot d_{50} / \nu_f$.

A log law for a hydraulically rough boundary is often employed to formulate a friction equation for the plane surface of an eroded granular bed. The law relates the interfacial friction coefficient λ_b with the hydraulic radius of discharge area associated with the top of the contact layer R_{hb} and the equivalent roughness of the top of the bed k_s,

$$\sqrt{\frac{8}{\lambda_b}} = \frac{1}{\kappa} \cdot \ln\left(\frac{B_s \cdot R_{hb}}{k_s}\right) \tag{2}$$

A determination of the bed roughness k_s in layered models requires an appropriate representation of bed-shear parameters and at the same time sufficient simplicity of the k_s-formula to avoid problems with the model numerical solution. Based on extensive testing, we suggested (Matoušek and Krupička, 2014) the formula that satisfied the contradicting requirements,

$$k_s / d_{50} = 1.35 \cdot W_{s*}^{0.5} \cdot \theta^{1.58} \tag{3}$$

where is the dimensionless grain parameter $W_{s*} = \sqrt[3]{(S_s - S_f)/(g \cdot \cos\omega \cdot \nu_f)} \cdot w_t \cdot \cos\omega$.

2. EXPERIMENT WITH STATIONARY BED IN INCLINED PIPE

Inclined flow experiments were carried out for slurry flow of sand-water mixture in the 100 mm loop with an inclinable inverted U-tube at Institute of Hydrodynamics in Prague. The loop, its measuring equipment, measuring techniques and experimental procedures for inclined flow tests are described in Matoušek et al. (2018).

The experiments were carried out with the same sand as the experiments reported in another paper in these proceedings (Matoušek et al., 2019), i.e. with the 0.55 mm sand (narrow graded sand with the mean grain size of 0.55 mm and grain density of 2597 kg/m^3). Measurements included mean flow velocity V_m, manometric pressure drops in ascending and descending limbs of the U-tube, the delivered concentration C_{vd} in the vertical section of the loop, and chord-averaged vertical concentration distributions in pressure-drop measuring sections of both limbs of the U-tube.

It was much easier to produce stationary bed condition in the ascending leg of the inverted U-tube than in the descending leg. At inclination angles steeper than 20 degrees it was virtually impossible to reach the stationary bed condition in the descending pipe. Also, due to much more intense accumulation of grains in the ascending leg of the U-tube the mean solids concentration in the descending leg was very low in case of flow above the stationary bed.

3. EVALUATION OF EFFECT OF INCLINATION ON BED TRANSPORT AND ROUGHNESS

In order to evaluate transport and friction conditions at the top of the eroded stationary bed, the bed shear condition must be determined from values of the collected experimental parameters.

3.1 PRESSURE DROP AND BED SHEAR STRESS

The bed shear stress, i.e. the mean shear stress at the top of the stationary bed, is related to the frictional part of the total pressure drop in the flow. There, this pressure drop must be determined from the measured manometric pressure drop,

$$\Delta p_{\omega}^{fric} = \Delta p_{\omega}^{man} - \Delta p_{\omega}^{static} = \Delta p_{\omega}^{man} - C_{a\omega}\left(\rho_s - \rho_f\right) gL \sin\omega \qquad (4)$$

Note that the static pressure drop includes $(\rho_s - \rho_f)$ because it is subtracted from the manometric (not absolute) pressure drop. Note also that the static pressure drop is associated with the mean concentration $C_{a\omega}$ of particles contributing to this part of the total pressure drop. If all particles above the bed contribute, then $C_{a\omega}$ equals to the mean concentration of solids in the flow area above the bed, $C_a = (C_{vi}A - C_b A_b)/(A - A_b)$. In all our cases with a stationary deposit, $C_{a\omega} < C_{vi}$ (the concentration in the entire pipe cross section). The experimental data for ascending flows gave very low (or even negative) values of Δp_{ω}^{fric} when C_{vi} was used instead of $C_{a\omega}$ in Equation 4. The bed shear stress is calculated from the frictional pressure drop,

$$\tau_b = \frac{\Delta p_{\omega}^{fric}}{L} R_{hb} = \rho_w g R_{hb} i_{\omega}^{fric} \qquad (5)$$

where R_{hb} is related to the part of the flow cross section area associated with the top of the bed (A_{ab} in Figure 1). This is determined from the experimental data using a method described elsewhere (Matoušek, 2011).

3.2 BED TRANSPORT (SOLIDS DISCHARGE)

The procedure described above, including Equations 4 & 5, enables to determine the bed Shields parameter θ from the experimental data provided that experimental values of Δp_{ω}^{fric}, V_m, C_{vi}, y_b, and C_b are known for a particular test run. Consequently, the solids transport formula (Equation 1) uses the determined value of θ to predict q_s and this is further re-arranged to predict C_{vd} ($C_{vd} = q_s \cdot O_b/(V_m \cdot A)$). The predicted C_{vd} can be compared with the measured value of C_{vd} to check the accuracy of the transport-formula prediction. This procedure has been frequently used for experiments in horizontal pipes.

We applied the procedure to our test results for the inclined flows and noticed that a predicted value of C_{vd} was extremely sensitive to the chosen values of y_b and $C_{a\omega}$. An experimental determination of y_b is associated with the highest uncertainty from all measured parameters. For $C_{a\omega}$, no direct experimental determination is possible.

Therefore, we decided to employ the transport formula in an evaluation of bed conditions in the observed inclined flows rather than to try to test its accuracy for inclined flows. For each run, we selected a combination of suitable values of y_b (estimated from the shape of the profile) and $C_{a\omega}$ (estimated from C_a) for which the transport formula predicted a C_{vd} value very similar to the measured one. For some cases, $C_{a\omega} = C_a$ was found appropriate, for other cases it had to be reduced ($C_{a\omega} < C_a$) in order to satisfy the C_{vd} prediction. Figures 2-4 show the measured concentration profiles and corresponding selected y_b positions for flow at two velocities V_m and various inclinations.

The procedure served to test the accuracy of the transport formula in flow at 0 degrees, where no static part had to be subtracted. The y_b-position at which the satisfactory agreement between the measured and predicted C_{vd} was reached corresponded with the position detected at the shape of the profile (see Figure 2, the right panel).

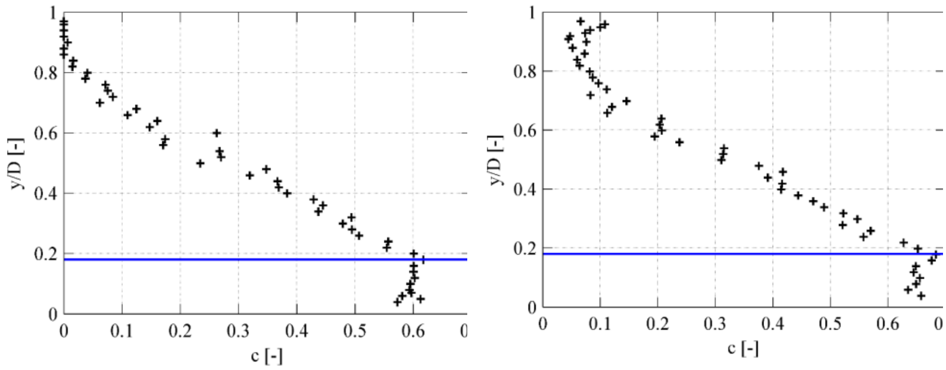

Figure 2. Measured concentration distribution and estimated position of top of stationary bed for flow at V_m = 1.5 m/s and inclination of -5 degree (left plot) and 0 degree (right plot).

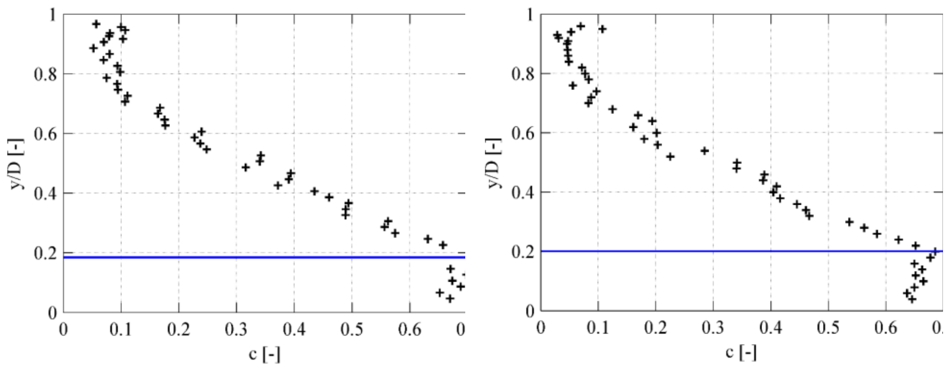

Figure 3. Measured concentration distribution and estimated position of top of stationary bed for flow at V_m = 1.5 m/s and inclination of +5 degree (left plot) and +15 degree (right plot).

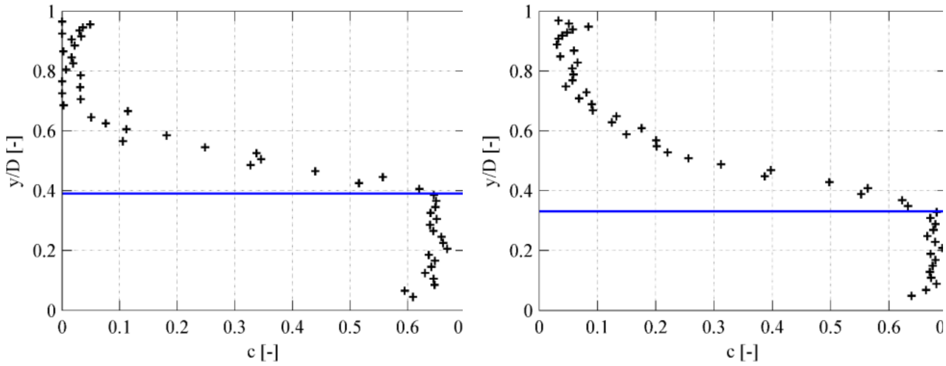

Figure 4. Measured concentration distribution and estimated position of top of stationary bed for flow at $V_m = 1.0$ m/s and inclination of +25 degree (left plot) and +35 degree (right plot).

The bed shear conditions obtained by the procedure are further used to determine the experimental roughness of the bed and to predict the equivalent roughness by Equation 3. The comparison is discussed in the next chapter.

3.3 BED ROUGHNESS

The equivalent roughness for the log law (Equation 2) of the top of the eroded bed is determined from experimental results processed by Equations (2, 4 & 5) and additional equations using the procedure described above. The experimentally determined roughness is compared with the predicted roughness from Equation 3 in Figure 5. The agreement is very reasonable for the entire range of available flow conditions, including a broad range of angles of flow inclination from -5 to +45 degree. Based on this comparison, it can be concluded that for the tested flows the effect of flow inclination is sufficiently captured in the formulae used in the layered model for inclined flows.

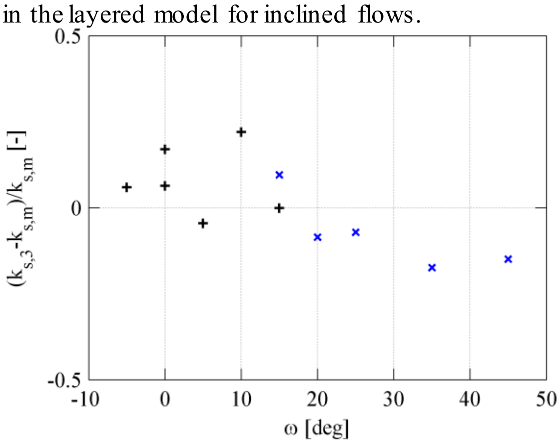

Figure 5. Relative deviation of bed roughness predicted by Equation 3 ($k_{s,3}$) from bed roughness obtained by experiment ($k_{s,m}$). Legend: + – flow at $V_m = 1.5$ m/s, x – flow at $V_m = 1.0$ m/s.

A more general conclusion must be based on a broader data set including flows of different delivered concentrations and different solids fractions. A collection of such a data set is work currently in progress.

4. CONCLUSIONS

The experimental results for flow of the 0.55 mm sand slurry above a stationary bed in a 100 mm pipe inclined to various angles between -5 to +45 degree were used to test the combination of the transport and friction formulae for their ability to capture the effect of flow inclination. The combination of the two formulae seems to satisfactorily describe the shear conditions at the top of the inclined eroded bed and hence their use in the layered model for inclined flows is justified, at least in the range of the tested conditions.

Our experimental results suggest that grains occupying the stationary bed do not contribute to the static pressure drop developed due to pipe inclination. This is different from grains in the sliding bed which contribute to the static pressure drop. Moreover, the experiments suggest that in a majority of cases not all grains in the shear layer above the stationary bed contribute to the static pressure drop either.

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